Data Representation

SARCAR Sayan

Faculty of Library, Information, and Media Science

筑波大学 University of Tsukuba

Contents

- What do we mean by data?
- How can data be represented electronically?
- What number systems are often used and why?
- How do number systems of different bases work?
- How do you convert a number between binary and decimal?

Data

– Many definitions are possible depending on context

- We will say that:
  – data is a physical representation of information

- Data can be stored
  – e.g.: computer disk, cash till

- Data can be transmitted
  – e.g.: fax

- Data can be processed
  – e.g.: cash till

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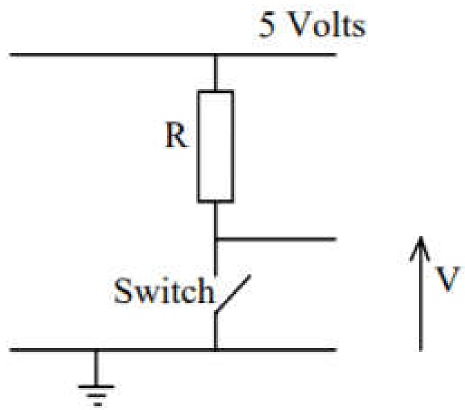
Electronic representation of data

- Information can be very complicated
  - e.g.:

    Numbers   Sounds

    Pictures   Codes
  - We need a simple electronic representation

- What can we do with electronics?
  - Set up voltages and currents
  - Change the voltages and currents



- A useful device is a switch
  - Switch Closed: V = 0 Volts
  - Switch Open: V = 5 Volts

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Representation of data

Information can be represented by a voltage level

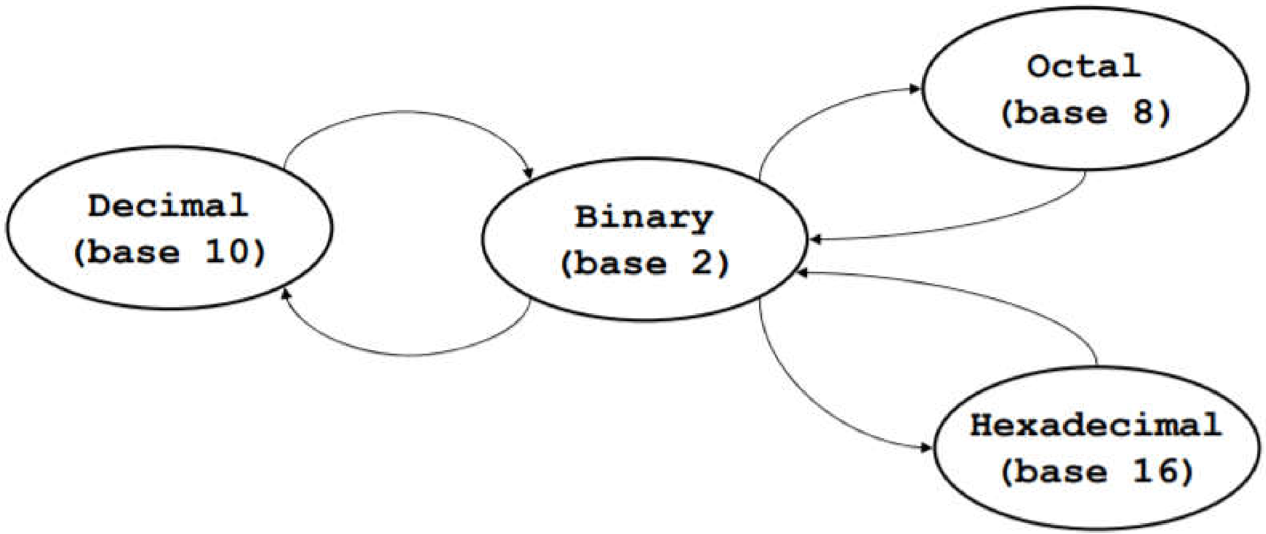
- The simplest information is TRUE/FALSE
  - This can be represented by two voltage levels:
    - 5 Volts for TRUE
    - 0 Volts for FALSE

- A voltage signal which has only two possibilities is a BIT
  - Bit stands for Binary Digit

- Binary means: only 2 possible values
  - False(0) True(1)

- Advantages of using binary representation
  - simple to implement in electronic hardware (switch)
  - good tolerance to noise

Number system overview



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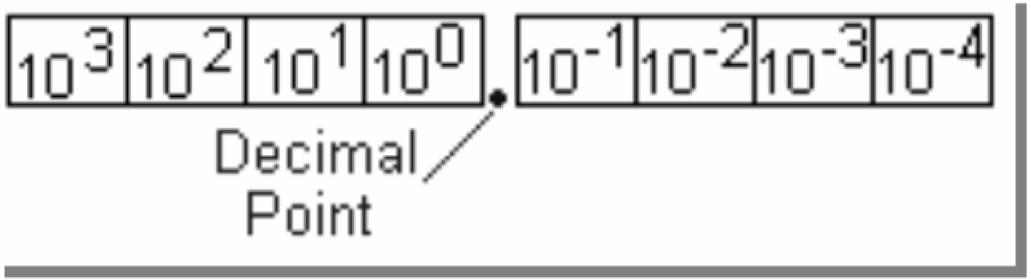
Decimal numbers

The decimal number system has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9

The decimal numbering system has a base of 10 with each position weighted by a factor of 10:



Binary numbers

• The binary number system has two digits: 0 and 1

• The binary numbering system has a base of 2 with each position weighted by a factor of 2:

POSITIVE POWERS OF TWO (WHOLE NUMBERS)									NEGATIVE POWERS OF TWO (FRACTIONAL NUMBER)					
2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
256	128	64	32	16	8	4	2	1	1/2	1/4	1/8	1/16	1/32	1/64
									0.5	0.25	0.125	0.0625	0.03125	0.015625

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Binary number system

Uses 2 symbols by our previous rule

– 0 and 1

Example: 10011 in binary is 1 x 2 + 1 x 2 + 1 x 2 =19

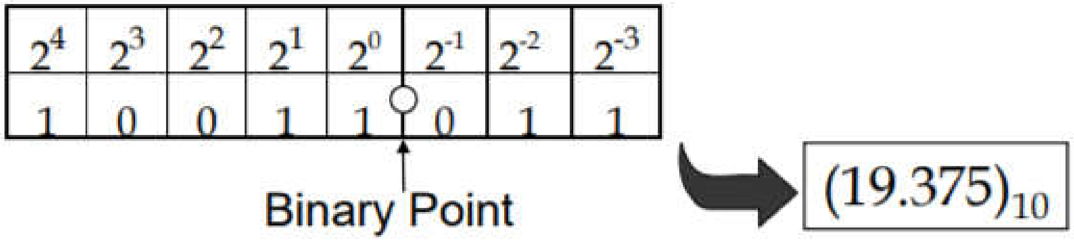
2^4	2^3	2^2	2^1	2^0
1	0	0	1	1

Binary is the base 2 number system

- Most common in digital electronics

Integer and Fractional parts

- Binary numbers can contain fractional parts as well as integer parts



- This 8-bit number is in Q3 format
  - 3 bits after the binary point

- How could 19.376 best be represented using an 8-bit binary number?
  - Quantization error

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Conversion- Decimal to Binary (1)

- The decimal number is simply expressed as a sum of powers of 2, and then 1s and 0s are written in the appropriate bit positions.

$$
\begin{aligned}
50_{10} &= 32 + 18 \\
&= 32 + 16 + 2 \\
&= 1\times 2^5 + 1\times 2^4 + 1\times 2^1 \\
50_{10} &= 110010_2
\end{aligned}
$$

$$
\begin{aligned}
346_{10} &= 256+90 \\
&= 256+64+26 \\
&= 256+64+16+10 \\
&= 256+64+16+8+2 \\
&= 1\times 2^8 + 1\times 2^6 + 1\times 2^4 + 1\times 2^3 + 1\times 2^1 \\
346_{10} &= 101011010_2
\end{aligned}
$$

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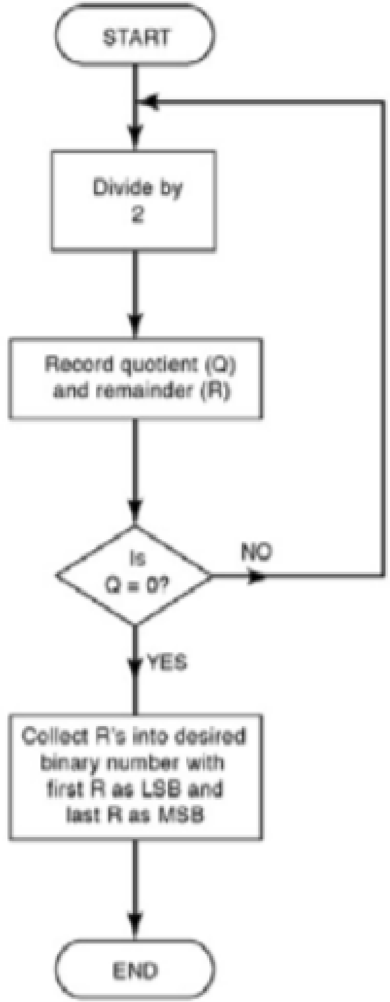
Conversion- Decimal to Binary (2)

- Repeated division

		quotient	remainder	
50/2	=	25	0	LSB
25/2	=	12	1	
12/2	=	6	0	
6/2	=	3	0	
3/2	=	1	1	
1/2	=	0	1	MSB

$$50_{10}=110010_2$$



Conversion: Binary tp Decimal

- The simplest way is to represent the binary number as
  $a_n \times 2^{n-1} + \ldots + a_2 \times 2^2 + a_1 \times 2^1 + a_0 \times 2^0$

- The conversion can be done by substituting the a's with the given bits then multiplying and adding:
  - eg: Convert $(1101)_2$ into decimal
  - $1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (13)_{10}$

- Other algorithms can be used as alternatives if you prefer

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Binary addition

- First recall decimal addition

	1	1	1	
A	1	2	3	4
+ B		9	8	7
Sum	2	2	2	1

- In binary addition we follow the same pattern but
  - 0 + 0 = 0 carry-out 0
  - 0 + 1 = 1 carry-out 0
  - 1 + 0 = 1 carry-out 0
  - 1 + 1 = 0 carry-out 1
  - 1 + 1 + carry-in = 1 carry-out 1

		1		
A	0	1	1	1
+ B	0	1	1	0
Sum	1	1	0	1

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Information Interaction Caveats

- Note that we need to consider 3 inputs per bit of binary number
  - A, B and carry-in

- Each bit of binary addition generates 2 outputs
  - sum and carry-out

Hexadecimal numbers

- Decimal, binary, and hexadecimal numbers

DECIMAL	BINARY	HEXADECIMAL
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

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Hexadecimal numbers conversions

Binary-to-hexadecimal conversion

1. Break the binary number into 4-bit groups
2. Replace each group with the hexadecimal equivalent

Hexadecimal-to-decimal conversion

1. Convert the hexadecimal to groups of 4-bit binary
2. Convert the binary to decimal

Decimal-to-hexadecimal conversion

– Repeated division by 16

Binary coded decimal (BCD)

- Use 4-bit binary to represent one decimal digit
- Easy conversion
- Wasting bits (4-bits can represent 16 different values, but only 10 values are used)
- Used extensively in financial applications

DECIMAL DIGIT	0	1	2	3	4	5	6	7	8	9
BCD	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

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Binary coded decimal (BCD)

- Convert 0110100000111001(BCD) to its decimal equivalent.

  0110 1000 0011 1001

  6 8 3 9

- Convert the BCD number 011111000001 to its decimal equivalent.

  0111 1100 0001

  7 ↑ 1

  The forbidden code group indicated an error

Putting it together

Decimal	Binary	Octal	Hexadecimal	BCD
0	0	0	0	0000
1	1	1	1	0001
2	10	2	2	0010
3	11	3	3	0011
4	100	4	4	0100
5	101	5	5	0101
6	110	6	6	0110
7	111	7	7	0111
8	1000	10	8	1000
9	1001	11	9	1001
10	1010	12	A	0001 0000
11	1011	13	B	0001 0001
12	1100	14	C	0001 0010
13	1101	15	D	0001 0011
14	1110	16	E	0001 0100
15	1111	17	F	0001 0101

Gray codes

- Only 1 bit changes in the count sequence
- Useful for industrial control

DECIMAL	BINARY	GRAY CODE
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001

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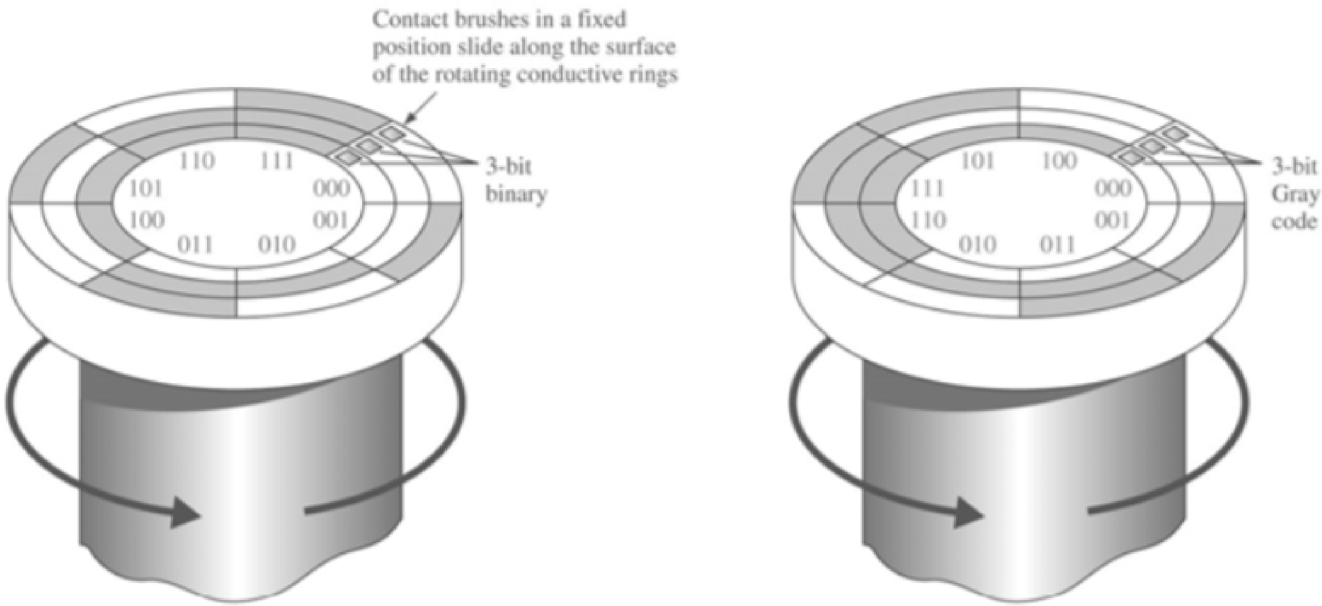
Gray codes

- Binary code results in glitches
- Gray code avoids glitches



(a) Binary

(b) Gray code

ASCII code

Codes representing letters of the alphabet, punctuation marks, and other special characters as well as numbers are called alphanumeric codes.

- The most widely used alphanumeric code is the American Standard Code for Information Interchange (ASCII). The ASCII (pronounced "askee") code is a seven-bit code.

Character	Seven-Bit ASCII	Octal	Hex	Character	Seven-Bit ASCII	Octal	Hex
A	100 0001	101	41	Y	101 1001	131	59
B	100 0010	102	42	Z	101 1010	132	5A
C	100 0011	103	43	0	011 0000	060	30
D	100 0100	104	44	1	011 0001	061	31
E	100 0101	105	45	2	011 0010	062	32
F	100 0110	106	46	3	011 0011	063	33
G	100 0111	107	47	4	011 0100	064	34
H	100 1000	110	48	5	011 0101	065	35
I	100 1001	111	49	6	011 0110	066	36
J	100 1010	112	4A	7	011 0111	067	37
K	100 1011	113	4B	8	011 1000	070	38
L	100 1100	114	4C	9	011 1001	071	39
M	100 1101	115	4D	blank	010 0000	040	20
N	100 1110	116	4E	.	010 1110	056	2E
O	100 1111	117	4F	(	010 1000	050	28
P	101 0000	120	50	+	010 1011	053	2B
Q	101 0001	121	51	$	010 0100	044	24
R	101 0010	122	52	*	010 1010	052	2A
S	101 0011	123	53	)	010 1001	051	29
T	101 0100	124	54	—	010 1101	055	2D
U	101 0101	125	55	/	010 1111	057	2F
V	101 0110	126	56	,	010 1100	054	2C
W	101 0111	127	57	=	011 1101	075	3D
X	101 1000	130	58	⟨RETURN⟩	000 1101	015	0D
				⟨LINEFEED⟩	000 1010	012	0A

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Questions to ponder

- How many different symbols can be represented with 4 bits?
- In a data transmission system the set of possible symbols is: {lower-case alphabet} U {upper-case alphabet} U {space, comma, full-stop} where 'U' denotes the 'union' of two sets. How many bits of information are needed for each symbol?
- In the above data transmission system the maximum transmission rate is 9600 bits per second. How long, in seconds, would it take to transmit the message:

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Home assignment

- Convert the following decimal numbers into binary. Do not use a calculator. a) 5 b) 99 c) 1024
- Convert the following binary numbers into decimal. Do not use a calculator. a) 1010 b) 10000000 c) 11111111
- Convert the following decimal numbers into hexadecimal. Do not use a calculator. a) 64 b) 98
- Convert the following hex numbers into binary directly without first converting them to decimal. Do not use a calculator. a) F8 b) 144
- Perform the following binary arithmetic: a) 00110111 + 00110010 b) 1100 + 0100 c) 00110100 - 00001010 d) 0010 - 0111

Q & A

Please write any feedback regarding class to

sayans@slis.tsukuba.ac.jp

Sub: Informatics class feedback

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